Data Structures and Algorithms

Lecture 9: Sorting

Outline of Today's Lecture

- Internal sorting
  - Three basic sorting algorithms ⟵ $\Theta(n^2)$
    - Insertion / bubble / selection sorts
  - One medium sorting algorithms ⟵ $\Theta(n^{1.5})$
    - Shell sort
  - Three fast sorting algorithms ⟵ $\Theta(n \log n)$
    - Merge / quick / heap sorts
  - Two special cases ⟵ $\Theta(n)$
    - Bin / radix sorts
- External sorting

Sorting

- **Motivation**: Suppose the record of student consists of student name, ID, course name, score, we sort *n* students by their scores.
- Given a set of records $r_1, r_2, \dots, r_n$ with key values $k_1, k_2, \dots, k_n$, the **Sorting Problem** is to arrange the records in non-decreasing order by their keys.
- Measures of algorithm cost:
  - Comparisons and Swaps are two main operations in sorting.

Sorting terminology

- Input is a set of records stored *in an array*.
- A sorting algorithm is **stable**, if it does not change the relative ordering of records with identical key values.
- **Internal sorting** *vs.* **External sorting**
  - In interval sorting, all records can be loaded into a computer memory
  - External sorting, there are too many records to be sorted → cannot be loaded into the memory

Internal Sorting Algorithms

- Three $\Theta(n^2)$ sorting algorithms
  - Insertion / bubble / selection sorts
- Shell sort -- $O(n^{1.5})$ in average case
- Three quick sorting algorithms-- $\Theta(n \log n)$
  - Merge / quick / heap sorts
- Two $\Theta(n)$ sorting algorithms for special cases of record keys
- Lower bounds for sorting

Insertion Sort (1)

- Assume you have sorted the first *i* (e.g., *i*=2) numbers, consider the (*i+1*)th number 36, insert the number in order so that the first *i+1* numbers are sorted.

Before insert:	**[27**	**53]**	**36**	**15**	**69**	**42**
After insert :	**[27**	**36**	**53]**	**15**	**69**	**42**

Insertion Sort (2)

- Traverse *i* from 1 to n-1, do the insertion

i=1: [53] 27 36 15 69 42

i=2: [27 53] 36 15 69 42

i=3: [27 36 53] 15 69 42

i=4: [15 27 36 53] 69 42

i=5: [15 27 36 53 69] 42

[15 27 36 42 53 69]

Insertion Sort (3)

```
template <class E>
void insertSort(E A[], int n) {
  for (int i=1; i<n; i++)
    for (int j=i; j>0 && A[j] < A[j-1]; j--)
      swap(A, j, j-1);
}
```

Best Case Analysis of Insertion Sort

- The best case occurs when the initial list of number are already sorted
- Best Case: 0 swap, n - 1 comparisons

15	**27**	**36**	**42**	**53**	**69**

Worst Case Analysis of Insertion Sort

- The worst case occurs when the initial list of number are reversely sorted
- At *i*-th iteration, performs *i* comparisons and swaps
- Total: $\Sigma i = n^2/2$ swaps and comparisons

69 53 42 36 27 15

Average Case Analysis of Insertion Sort

- At *i*-th iteration, performs *i/2* comparisons and swaps on average
- Total: $\Sigma i/2 = n^2/4$ swaps and comparisons

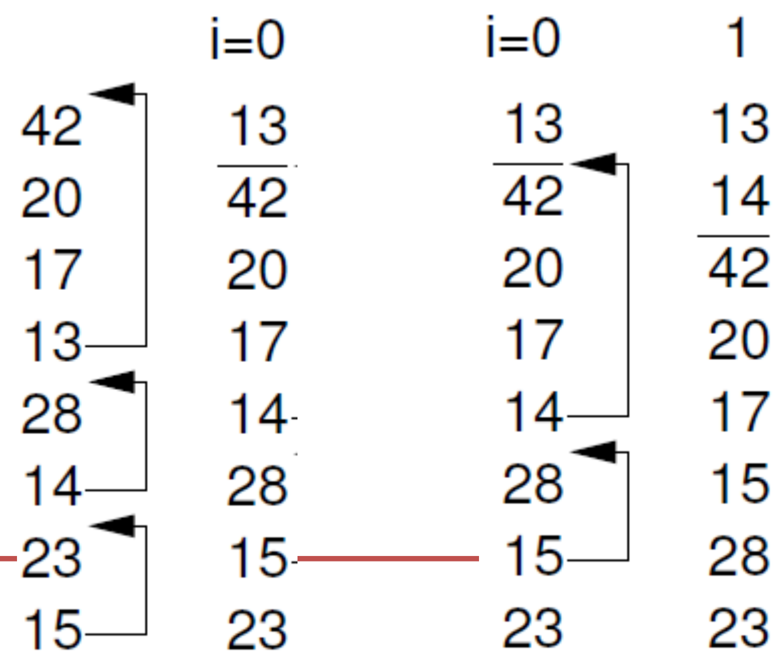
Before insert:	**[27**	**53]**	**36**	**15**	**69**	**42**
After insert :	**[27**	**36**	**53]**	**15**	**69**	**42**

Insertion Sort

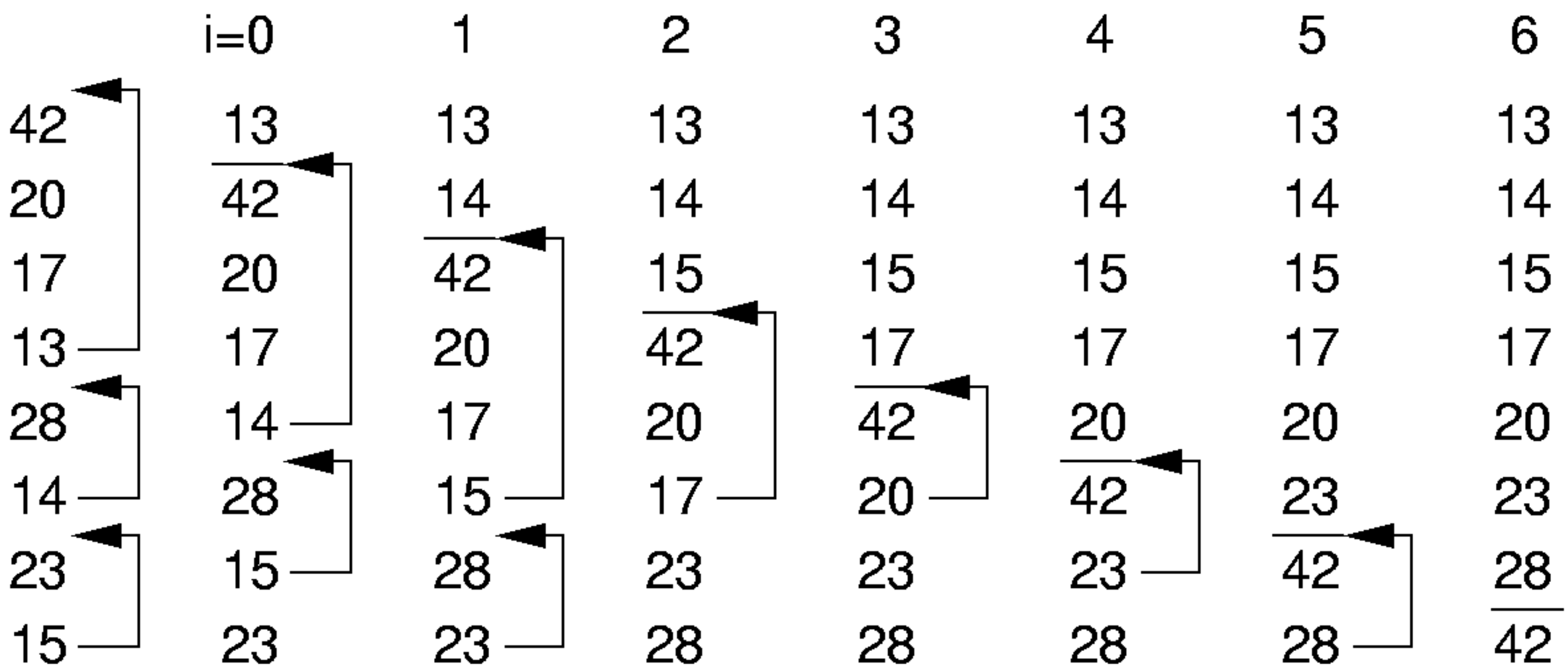
- Best Case: 0 swap, n - 1 comparisons
- Worst Case: $n^2/2$ swaps and comparisons
- Average Case: $n^2/4$ swaps and comparisons
- Insertion Sort is suitable for the cases where the records in the input array are almost sorted, e.g.,
  - Many records are already been sorted initially, but some a few new records are added

Bubble Sort (1)

- Scan from the bottom to the top, compare each adjacent values K[j-1] and K[j], swap them if the K[j] < K[j-1]. After the scan, the *smallest* value is at the top (bubble up)
- Do the 2nd scan from the bottom to the top-2
- …



Bubble Sort (2)



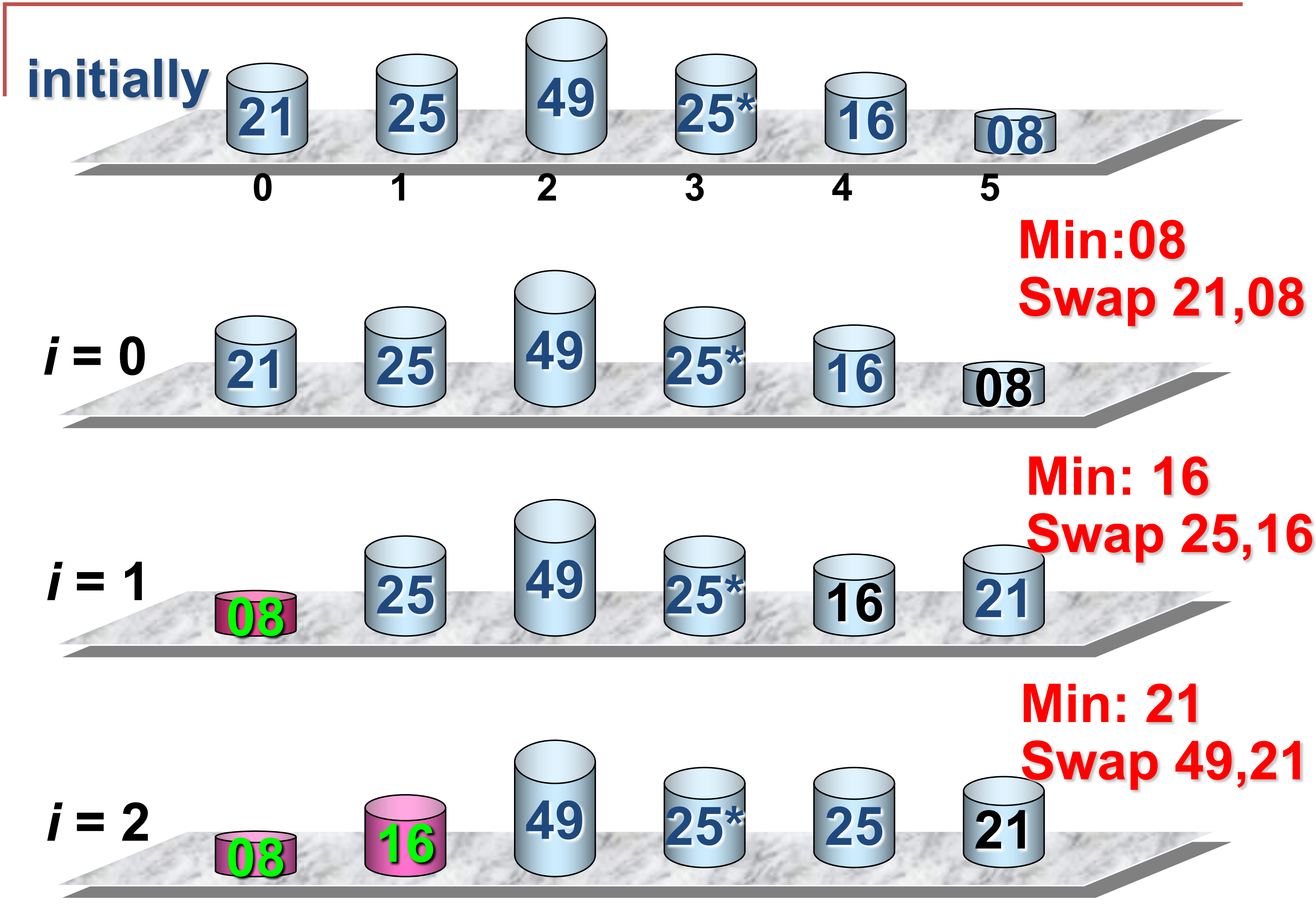
Bubble Sort (3)

```
template <class E>
void bubbleSort(E A[], int n) {
  for (int i=0; i<n-1; i++)
    for (int j=n-1; j>i; j--)
      if ( A[j] < A[j-1] )
        swap(A, j, j-1);
}
```

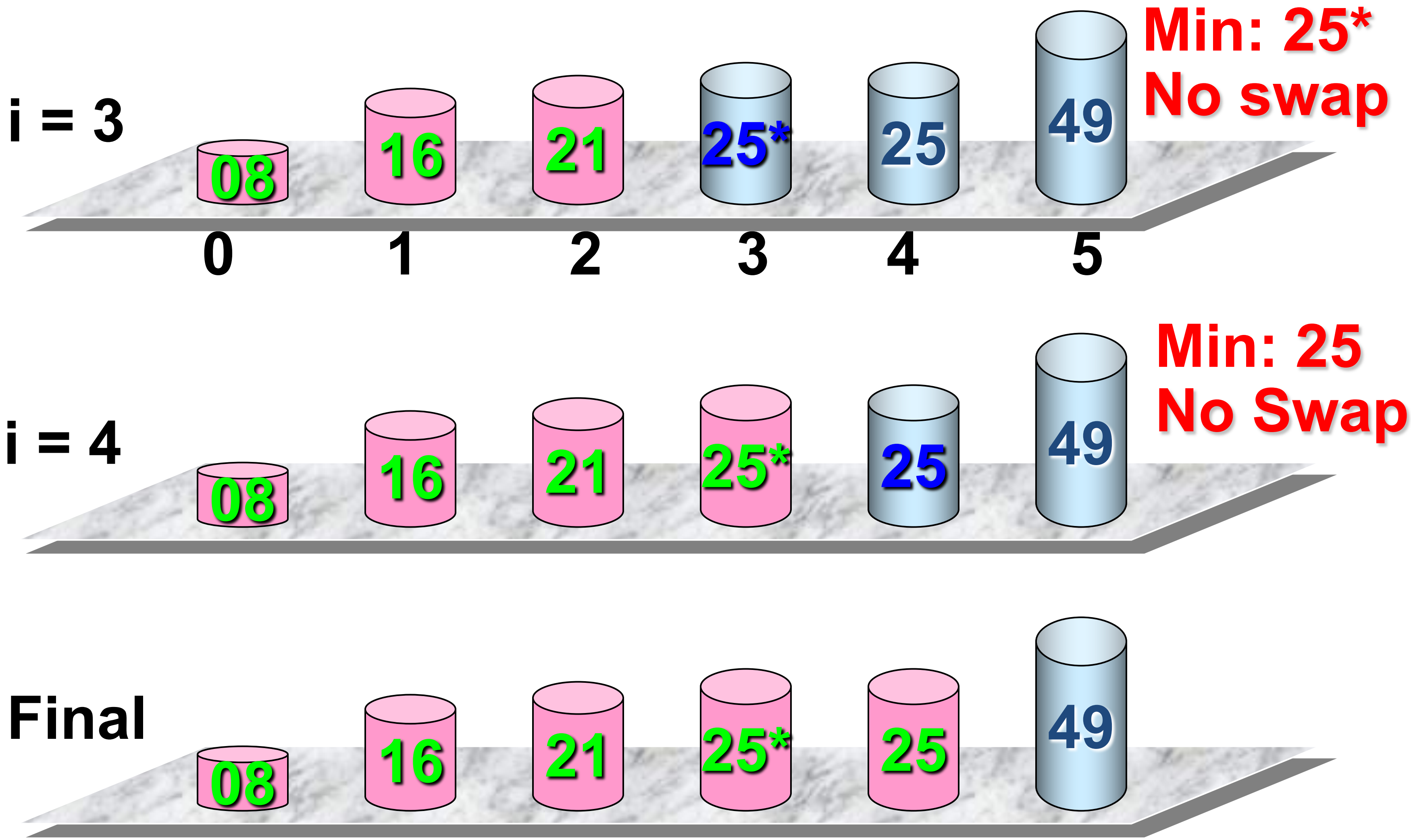
- Best Case: 0 swaps, $n^2/2$ comparisons
- Worst Case: $n^2/2$ swaps and comparisons
- Average Case: $n^2/4$ swaps and $n^2/2$ comparisons

Selection Sort

- Basic idea:
- First, select the smallest value, store it at the first location in the array
- Select the 2nd smallest value, store it at the 2nd location in the array
- …
- The array is sorted after *n* iterations



initially
21 25 49 25* 16 08
0 1 2 3 4 5
Min:08
Swap 21,08
i = 0
21 25 49 25* 16 08
Min: 16
Swap 25,16
i = 1
08 25 49 25* 16 21
Min: 21
Swap 49,21
i = 2
08 16 49 25* 25 21



i = 3
08
16
21
25*
25
49
Min: 25*
No swap
0
1
2
3
4
5
i = 4
08
16
21
25*
25
49
Min: 25
No Swap
Final
08
16
21
25*
25
49

Selection Sort (2)

```
template <class E>
void selectionSort(E A[], int n) {
  for (int i=0; i<n-1; i++) {
    int lowindex = i; // Remember its index
    for (int j=n-1; j>i; j--) // Find least
      if ( A[j] < A[ lowindex ])
        lowindex = j; // Put it in place
    swap(A, i, lowindex);
  }
}
```

- Best case: n-1 swaps, $n^2/2$ comparisons.
- Worst case: $n - 1$ swaps and $n^2/2$ comparisons.
- Average case: n-1 swaps and $n^2/2$ comparisons.

Summary of three $\Theta(n^2)$ sorting algorithms

	Insertion	Bubble	Selection
Comparisons:			
Best Case	$\Theta(n)$	$\Theta(n^2)$	$\Theta(n^2)$
Average Case	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Worst Case	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$
Swaps:			
Best Case	0	0	$\Theta(n)$
Average Case	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$
Worst Case	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$

Running time comparisons (n=100k)

- Random input

```
100000 numbers to be sorted:
Sort with InsertionSort, Time consumed:  1.438 (seconds)
Sort with bubble sort, Time consumed:    17.360 (seconds)
Sort with SelectionSort, Time consumed:  3.813 (seconds)
```

- The input is already sorted

```
100000 numbers to be sorted:
Sort with InsertionSort, Time consumed:  0.000 (seconds)
Sort with bubble sort, Time consumed:    3.766 (seconds)
Sort with SelectionSort, Time consumed:  3.905 (seconds)
```

- The input is reversely sorted

```
100000 numbers to be sorted:
Sort with InsertionSort, Time consumed:  2.984 (seconds)
Sort with bubble sort, Time consumed:    9.692 (seconds)
Sort with SelectionSort, Time consumed:  3.872 (seconds)
```

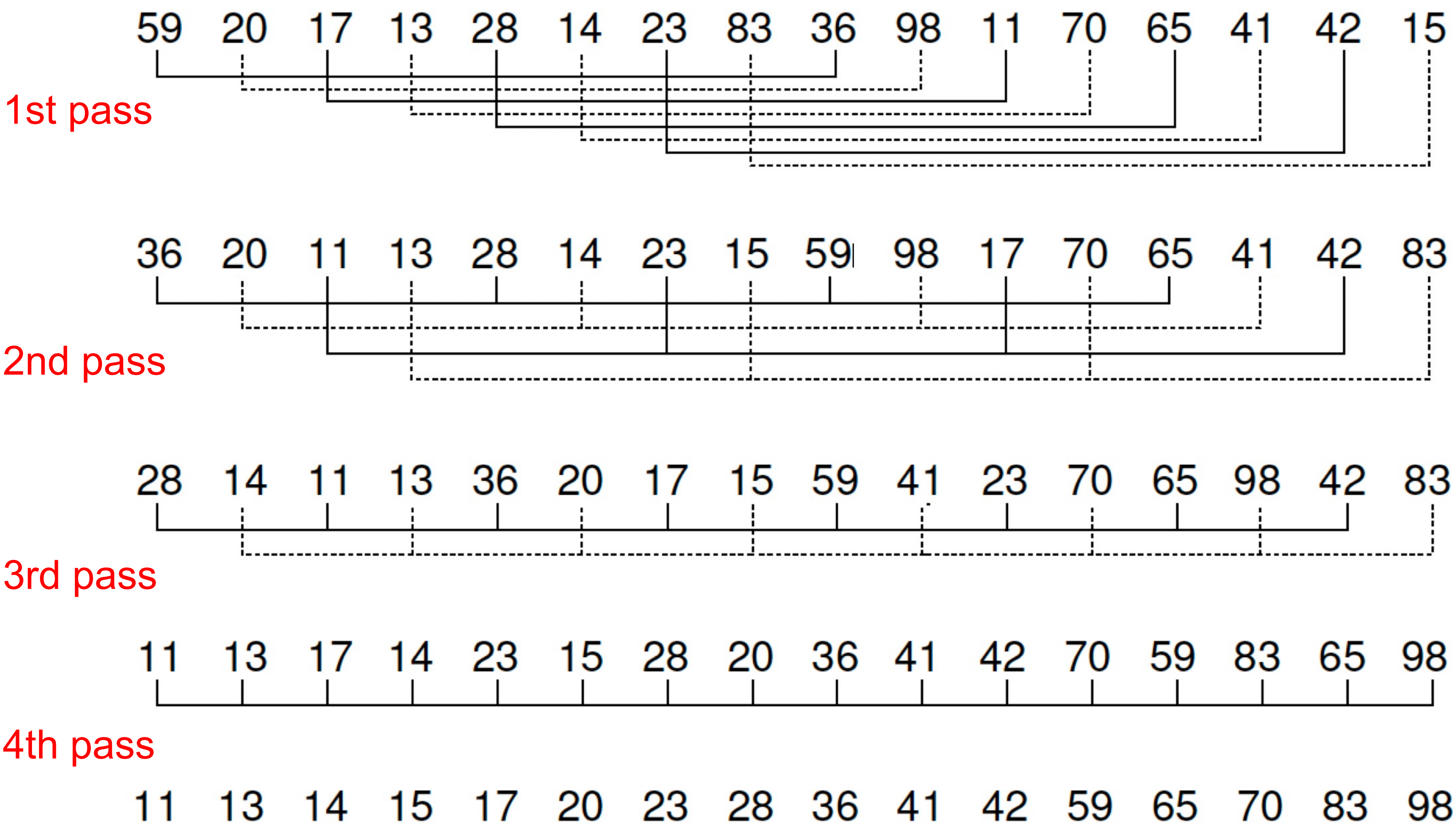
Shell sort

- Shellsort, named after its invertor, D.L. Shell.
  - Sometimes called the diminishing increment sort.
  - $O(n^{1.5})$ on average-case
- Its strategy is to make the list "mostly sorted" so that a final Insertion Sort can finish the job.
- Main steps:
  - Break the list into sublists
  - Sort them
  - Then, recombine the sublists

Shell sort process

- During each iteration/pass, Shellsort breaks the list into disjoint sublists so that each element in a sublist is a fixed number of postions aparts. e.g.,
  - Let us assume for convenience that *n*, the number of values to be sorted, is *a power of two*.
  - Shellsort will begin by breaking the list into *n/2* sublists of 2 elements each, where the array index of the 2 elements in each sublist differs by *n/2*.

Shell sort - an example (1)



Shell sort - an example (2)

- Some choices for increments would make Shellsort run more efficiently.
  - In particular, the choice of increments described above $(2^k, 2^{k-1}, \dots, 2,1)$ turns out to be relatively inefficient.
  - A better choice is the following series based on devision by three: $(\dots, 121,40,13,4,1)$.

Shellsort Implementation

```
//  Modified version of Insertion Sort for varying increments
template <typename E, typename Comp>
void inssort2(E A[], int n, int incr) {
  for (int i=incr; i<n; i+=incr)
    for (int j=i; (j>=incr) &&
          (Comp::prior(A[j], A[j-incr])); j-=incr)
      swap(A, j, j-incr);
}

template <typename E, typename Comp>
void shellsort(E A[], int n) { //  Shellsort
  for (int i=n/2; i>2; i/=2) //  For each increment
    for (int j=0; j<i; j++) //  Sort each sublist
      inssort2<E,Comp>(&A[j], n-j, i);
  inssort2<E,Comp>(A, n, 1);
}
```

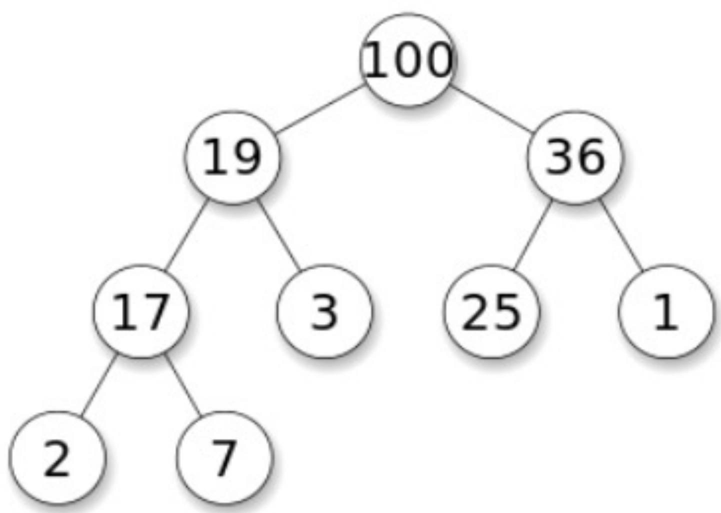
Three fast sorting algorithms

- Heap sort
  - $\Theta(n \log n)$ for the worst, best, average cases
- Merge sort
  - $\Theta(n \log n)$ for the worst, best, average cases
- Quick sort
  - $\Theta(n \log n)$ for the best and average cases
  - $\Theta(n^2)$ for the worst case

Heap – a special binary tree (Ch. II.5)

Heap: Complete binary tree with the heap property:

- Max-heap: each value in a node is no less than its children values
- The values in the tree are partially ordered.
  - The left child may less or greater than its right child

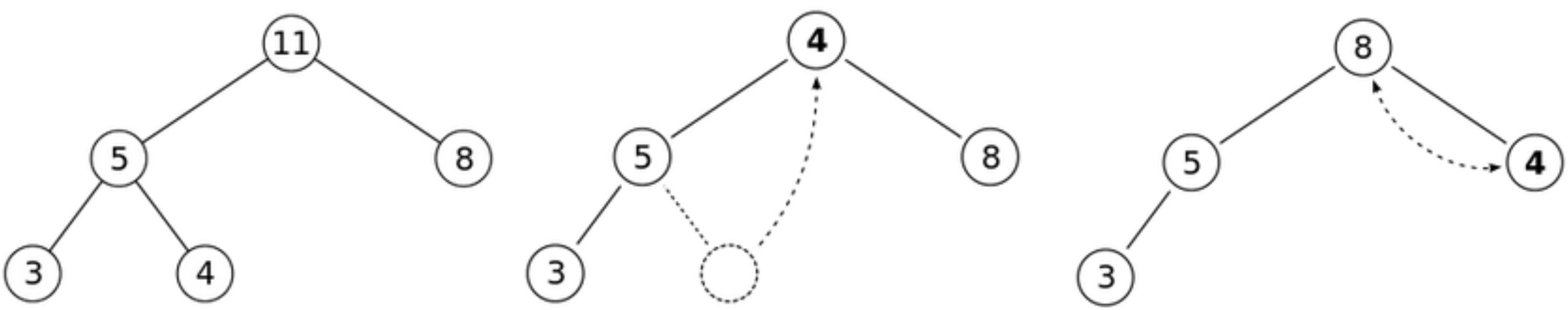


Heap Sort

- Given an array, build a max-heap
- Remove the maximum number from the heap
- Remove the next maximum number
- …
- Continue until no numbers are left in the heap

Heap -- removeMax

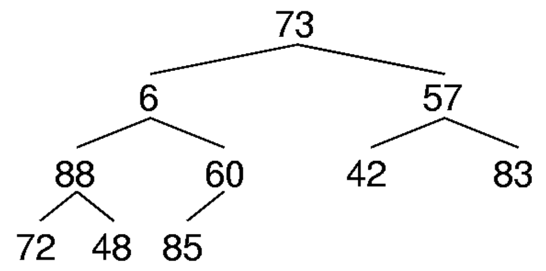
- Replace the root of the heap with the last element on the last level
- Compare the new root with its children (shift down operation)
  - if the new root is larger than its children, stop.
  - If not, swap the element with its largest children, and return to the previous step
  - Worst time complexity Θ(log n)



HeapSort Example (1)

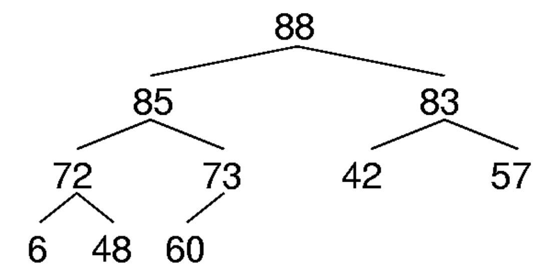
Original Numbers

73	6	57	88	60	42	83	72	48	85



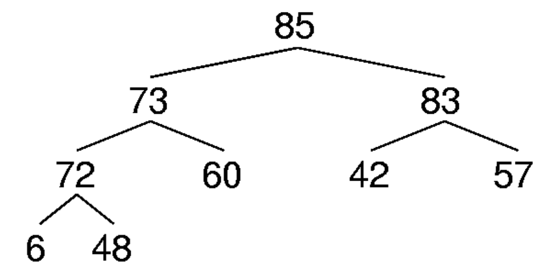
Build Heap

88	85	83	72	73	42	57	6	48	60



Remove 88

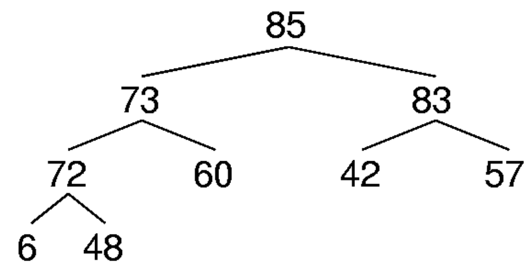
85	73	83	72	60	42	57	6	48	88



HeapSort Example (2)

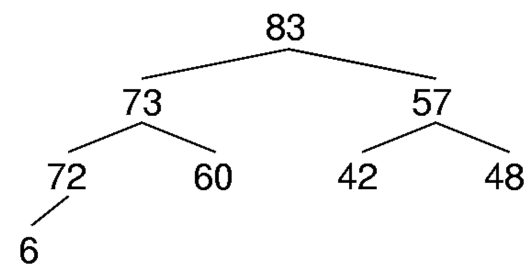
Remove 88

85	73	83	72	60	42	57	6	48	88



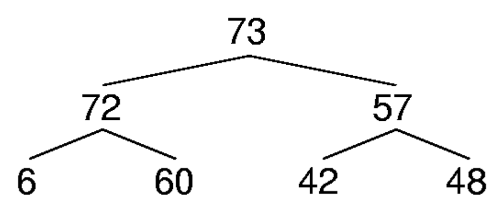
Remove 85

83	73	57	72	60	42	48	6	85	88



Remove 83

73	72	57	6	60	42	48	83	85	88



Heapsort

```
template <class E>
void heapSort(E A[], int n) { // Heapsort
  E mval;
  maxheap<E> H(A, n, n);
  for (int i=0; i<n; i++)   // Now sort
    H.removemax(mval);      // Put max at end
}
```

Analysis of Heap Sort

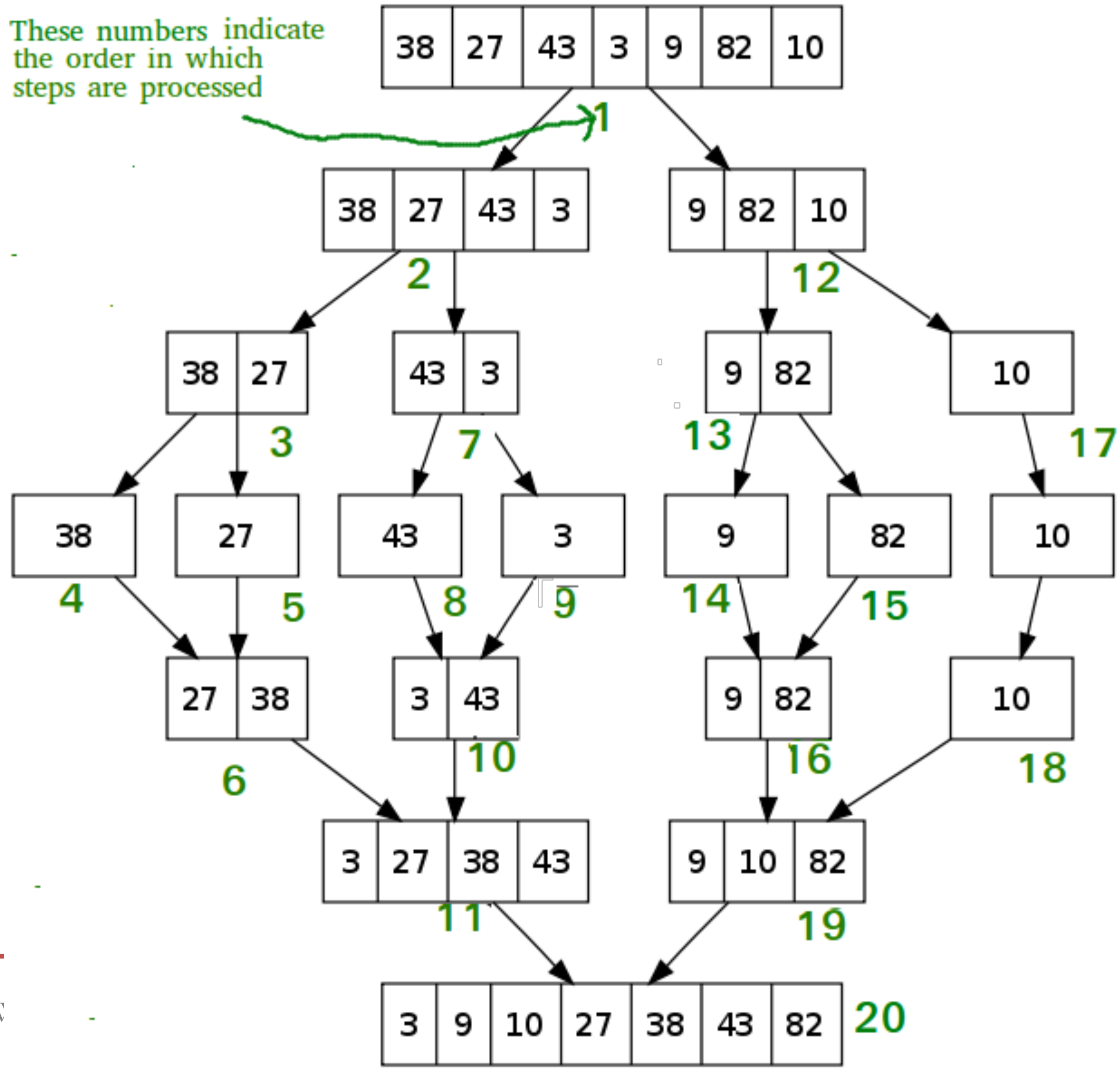
- Build a heap takes time $\Theta(n)$
- Remove the maximum value takes $\Theta(\log n)$, as heap is a complete tree
- Total time is $\Theta(n) + n\,\Theta(\log n) = \Theta(n \log n)$

Merge Sort

- Basic idea: divide and conquer

1. Given a list of numbers to be sorted
2. Split the list into two sub-lists with the identical length
3. Recursively sort the sub-lists, respectively
4. Merge the two sorted sub-lists

Merge Sort


These numbers indicate the order in which steps are processed
38 27 43 3 9 82 10
1
38 27 43 3
9 82 10
2
12
38 27
43 3
9 82
10
3
7
13
17
38
27
43
3
9
82
10
4
5
8
9
14
15
27 38
3 43
9 82
10
6
10
16
18
3 27 38 43
9 10 82
11
19
3 9 10 27 38 43 82
20

Merge sort with an array-based list (1)

- An array A[left, …,right], with the index range: left -- right
- How to split ?
- Let mid = (left + right)/2
- Left sub-list = A[left,…,mid]
- Right sub-list=A[mid+1,…, right]

Merge Sort with an array-based list (2)

- How to merge two sorted sub-lists A[left,…,mid], A[mid+1, …, right] ?

- An extra array temp[left…, right] is needed

- Step 1: move the smallest value of the first numbers of the two-sublists to array temp
  - If one sub-list is exhausted, just move the first number of the other sublist
- Continue until no numbers are left
- Copy back: A[left,…, right]=temp[left,…,right]

Merge Sort Implementation

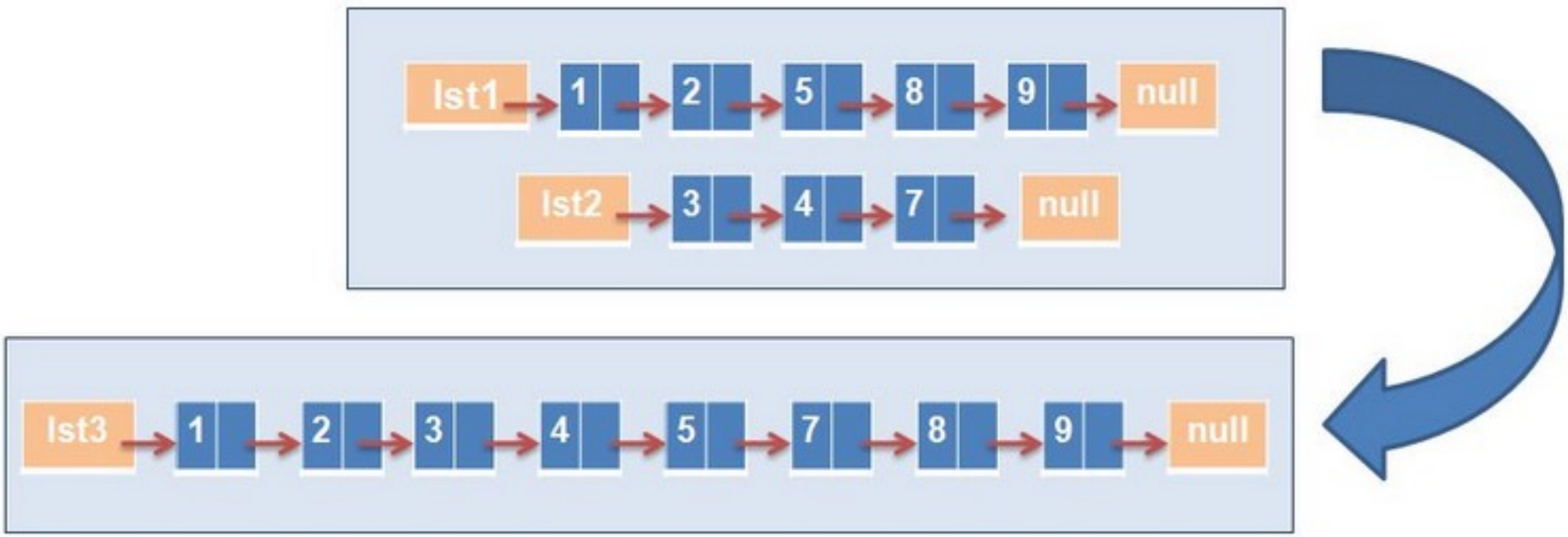
```
template <class E>
void mergeSort(E A[], E temp[],
                     int left, int right) {
  if (left == right) return;
  int mid = (left+right)/2;
  mergesort<E>(A, temp, left, mid);
  mergesort<E>(A, temp, mid+1, right);
  //merge two sorted sublists
  int i1 = left; int i2 = mid + 1;
  for (int curr=left; curr<=right; curr++) {
    if (i1 == mid+1)       // Left exhausted
      temp[curr] = A[i2++];
    else if (i2 > right)   // Right exhausted
      temp[curr] = A[i1++];
    else if (A[i1] < A[i2])
      temp[curr] = A[i1++];
    else temp[curr] = A[i2++];
  }
  for (int i=left; i<=right; i++) // Copy back
    A[i] = temp[i];
}
```

Merge Sort based on a linked list (1)

- How to split ?
- Given a singly linked list of numbers
- Need to scan half of numbers in the list

How to merge two sorted sub-lists ?

- Similar to the array-based version
- But no extra memory is needed



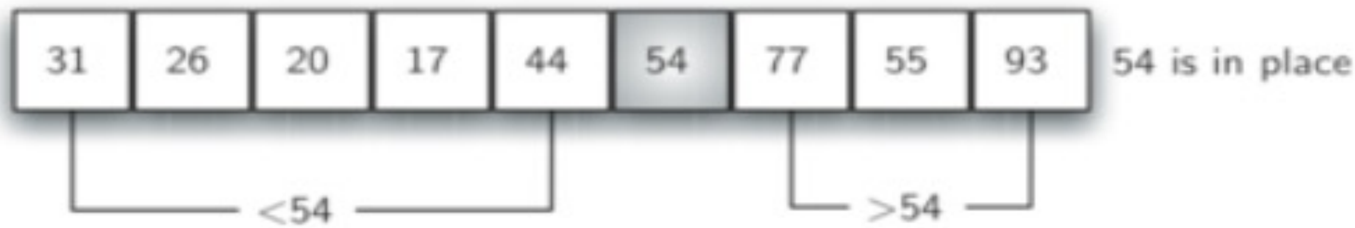
Time complexity of Merge Sort

- Let $T(n)$ be the running time for **n** numbers
- Split: $\Theta(1)$ for the array-based list
- Recursively sorting two sub-lists: $2 * T(n/2)$
- Merge: $\Theta(n)$
- $T(n)= 2\,T(n/2) + \Theta(n)$
- Expand the recurrence relationship, we have:
- $T(n)= \Theta(n \log n)$

Quick Sort

- Given an array of numbers A[left, …, right]
- Pick a value in the array as a pivot

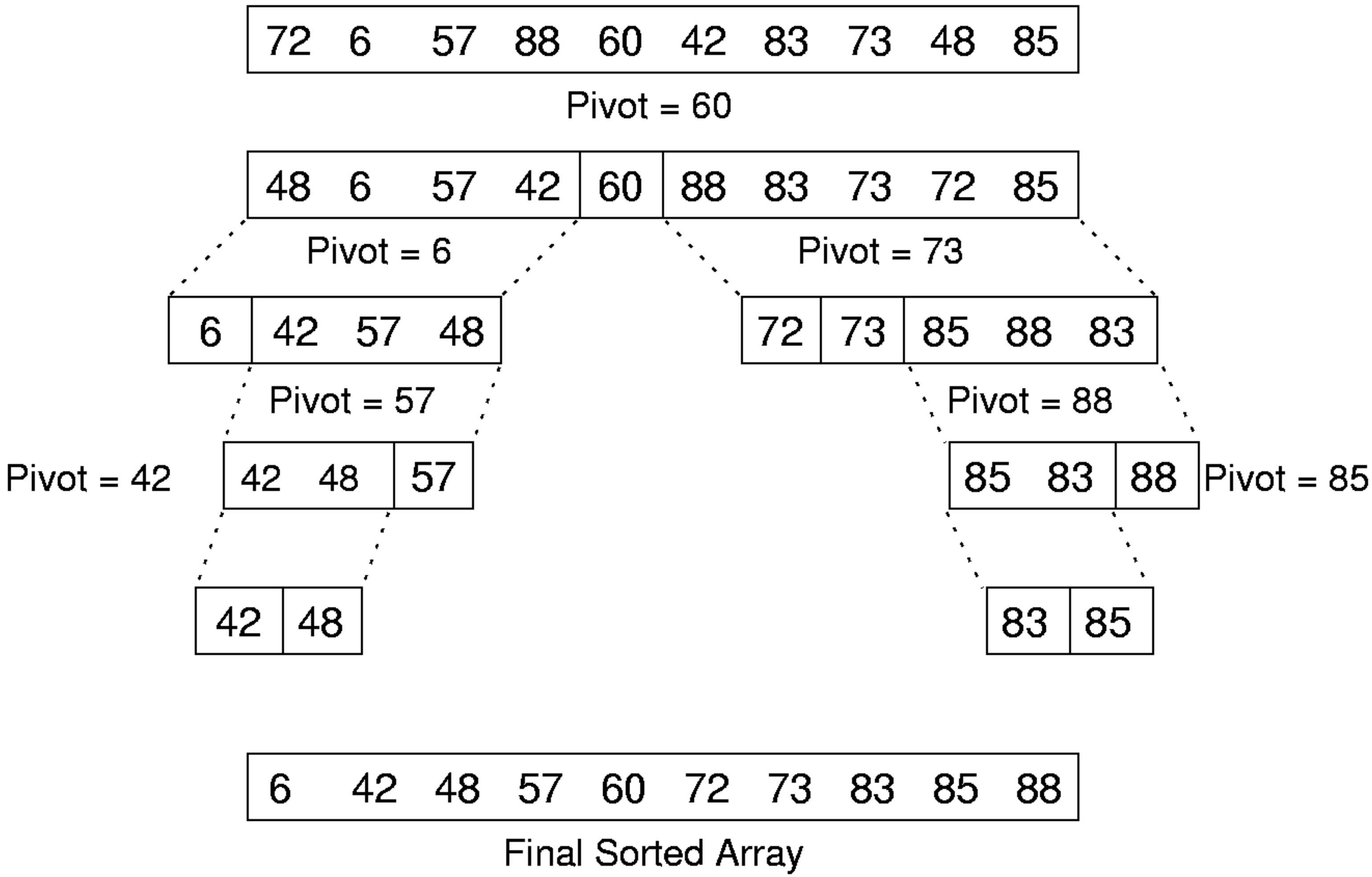
- Partition the array into three parts



1. The numbers in the left part are < pivot 54
2. The pivot itself in place
3. The numbers in the right part are ≥ pivot 54

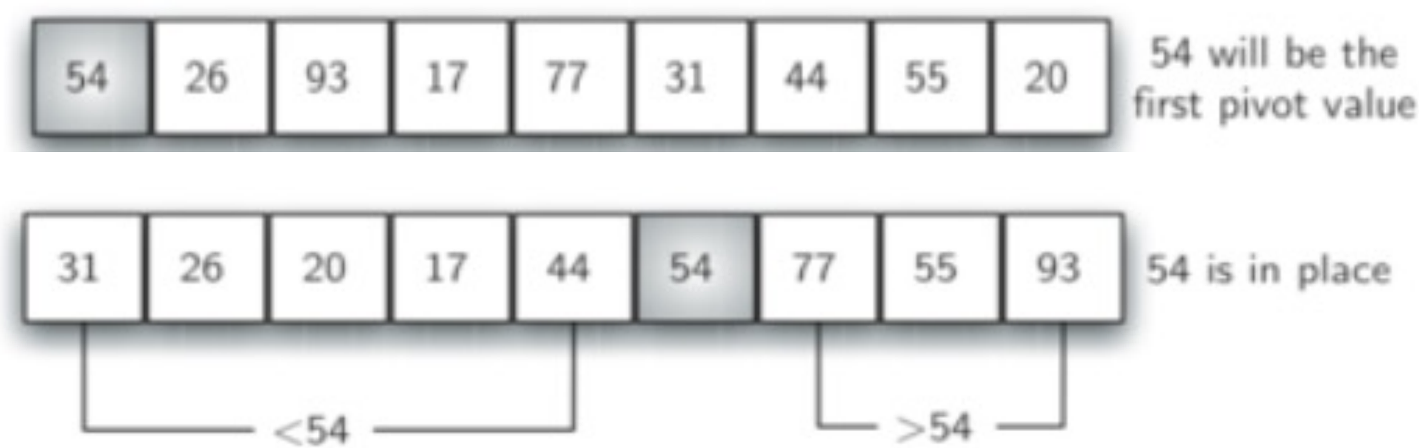
- Recursively sort the left and right parts

QuickSort Example



Two key problems in Quick Sort

- How to choose the pivot, such that the left and right parts are roughly balanced ?
  - The number of records in the left part is more or less the number in the right part
- How to efficiently partition an array by the pivot?



Solutions to the choice of a pivot

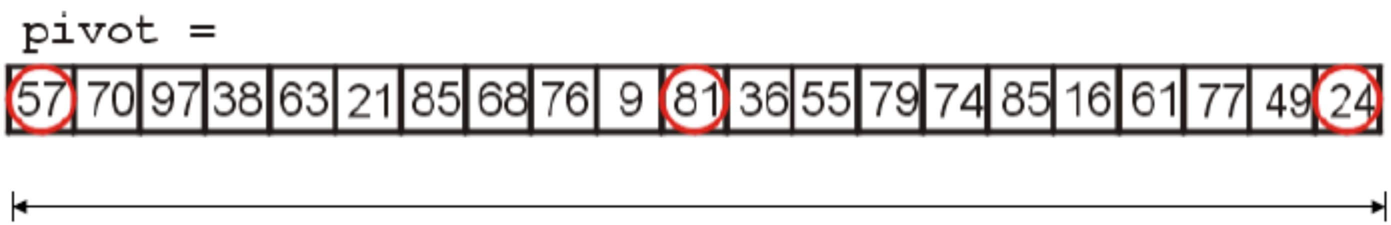
1. Traditionally, choose the first or the last number in the array
   - This is bad if the given array are already (or nearly) sorted, or reversely sorted, one part has 0 number, the other part has (n-1) numbers
2. Choose the middle number
   - mid = (left+right)/2; pivot = A[mid];
   - a better choice
3. median of three
   - Choose the pivot as the median of the first, middle and last numbers
   - Much better

Solutions to the choice of a pivot

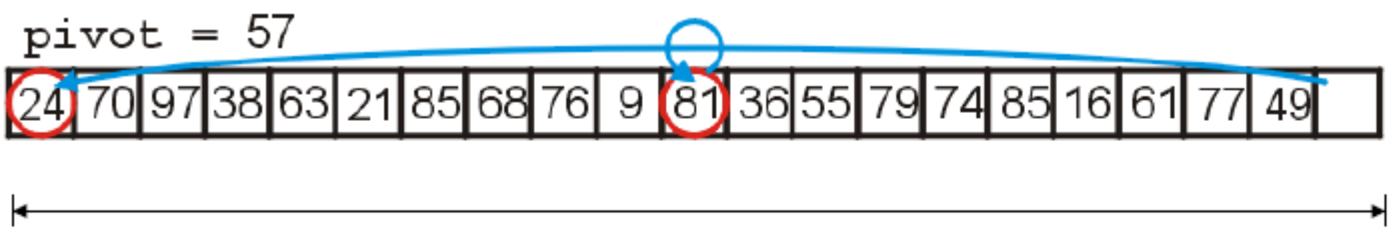
4. Randomly choose a number as the pivot
   - It is unlikely that a randomly chosen number is the smallest or the largest ones
   - Can combine with the 3rd solution, i.e., randomly choose three numbers, and select the median of the three as the pivot

Partition an array by a given pivot

- Assume that the pivot is the median of the first, middle, and last numbers

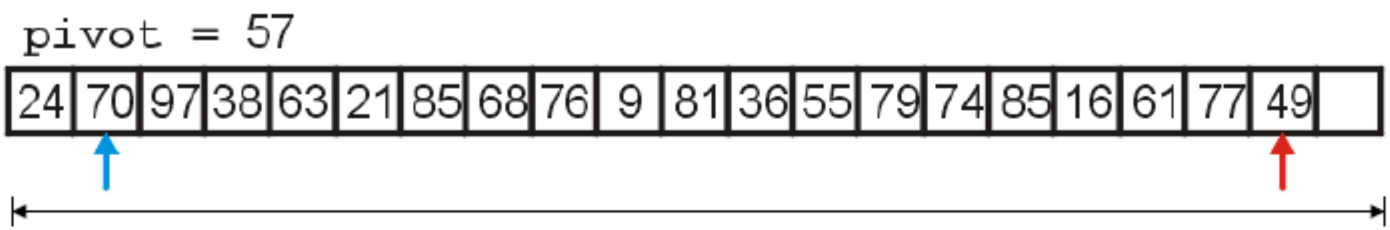


- First swap the pivot with the last number

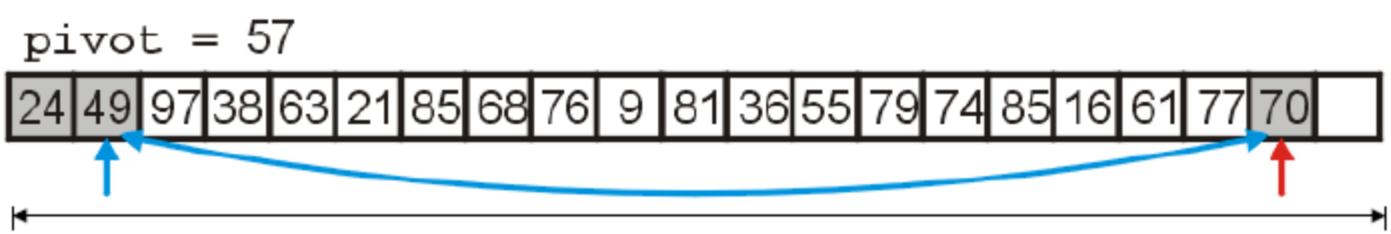


Partition an array by a given pivot

1. Start from the 1st location, search forward until we find a value ≥ pivot, e.g., 70 > 57
2. Start from the 2nd last location, search backward until we find a value < pivot, e.g., 49 < 57

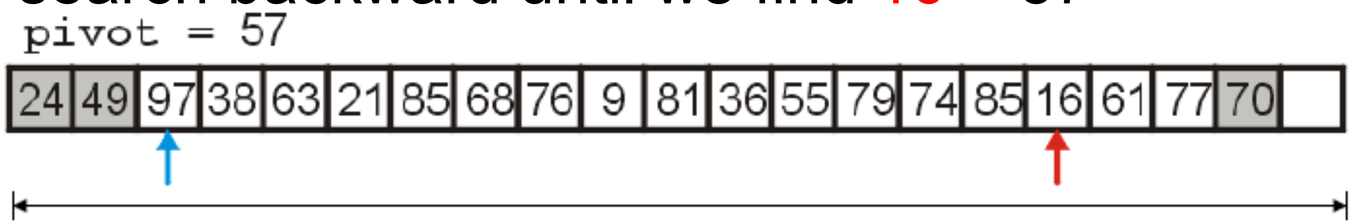


3. 70 and 49 are out of order, swap them

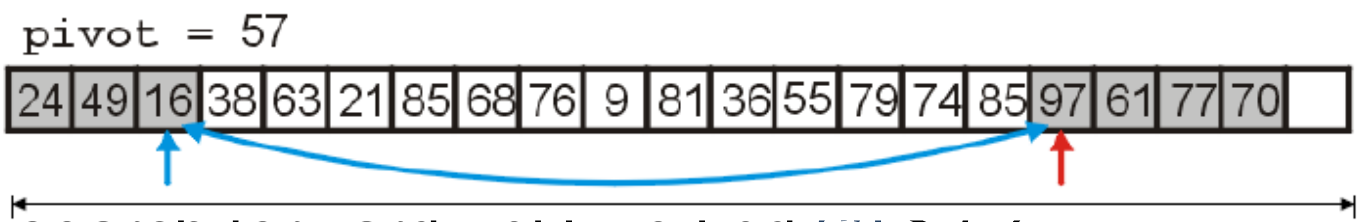


- We continue step1—step 3, see the next slides

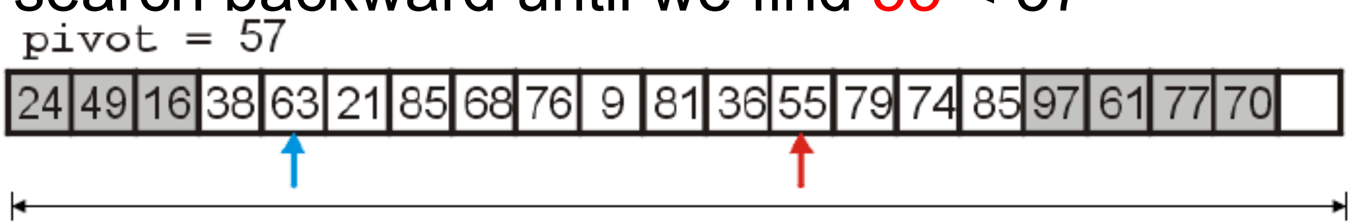
4. search forward until we find 97 ≥ 57
5. search backward until we find 16 < 57



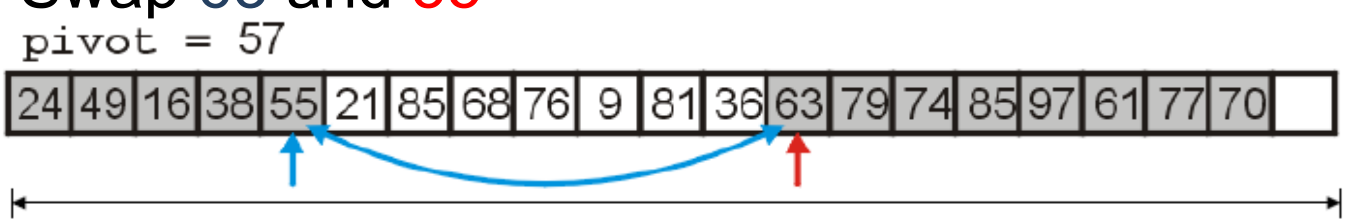
6. Swap 97 and 16



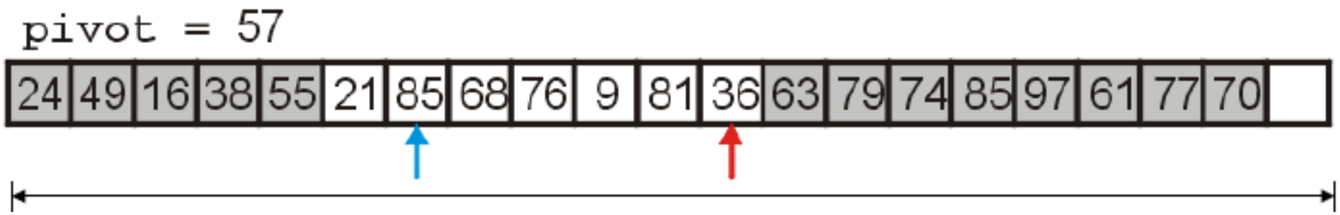
7. search forward until we find 63 ≥ 57
8. search backward until we find 55 < 57



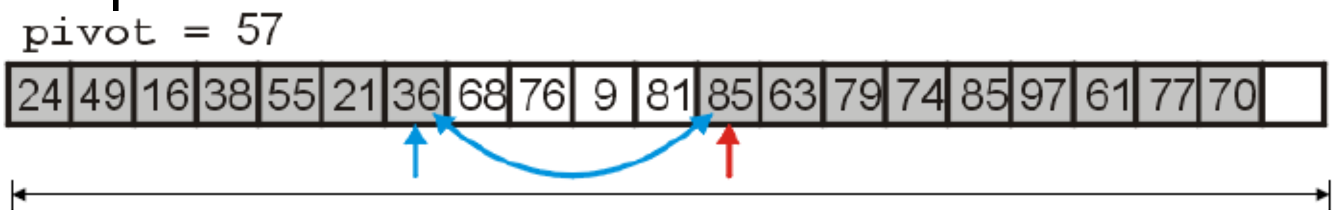
9. Swap 63 and 55



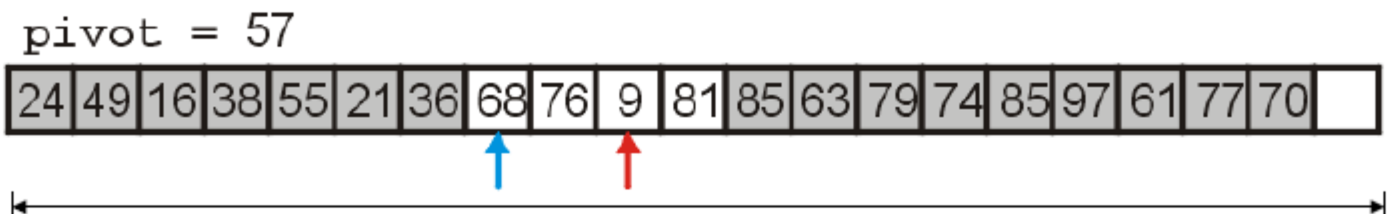
10. search forward until we find 85 ≥ 57
11. search backward until we find 36 < 57



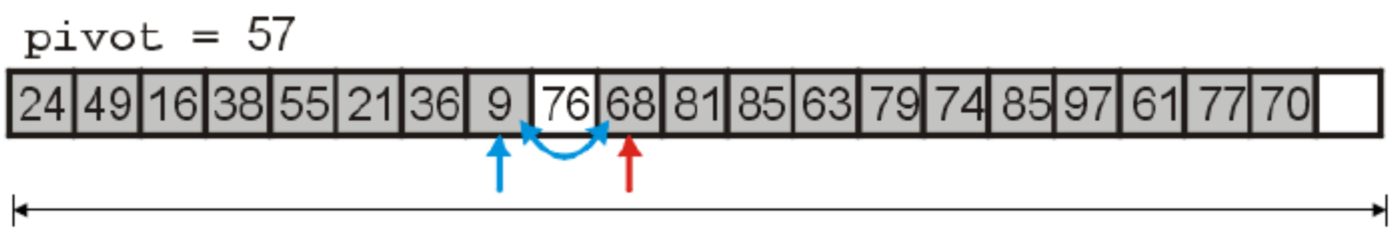
12. Swap 85 and 36



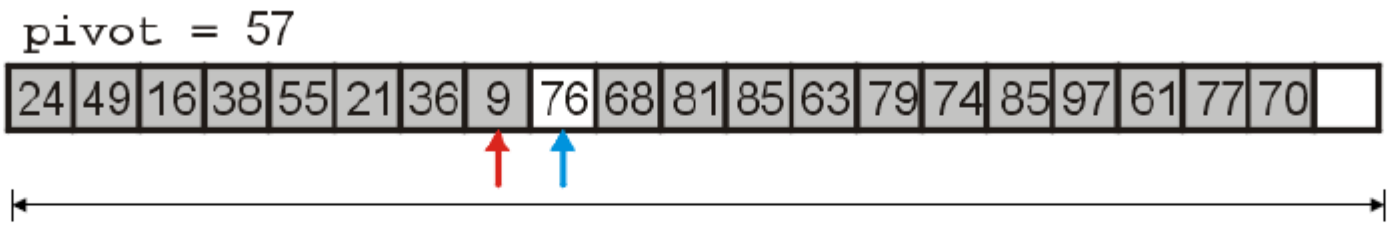
13. search forward until we find 68 ≥ 57
14. We search backward until we find 9 < 57



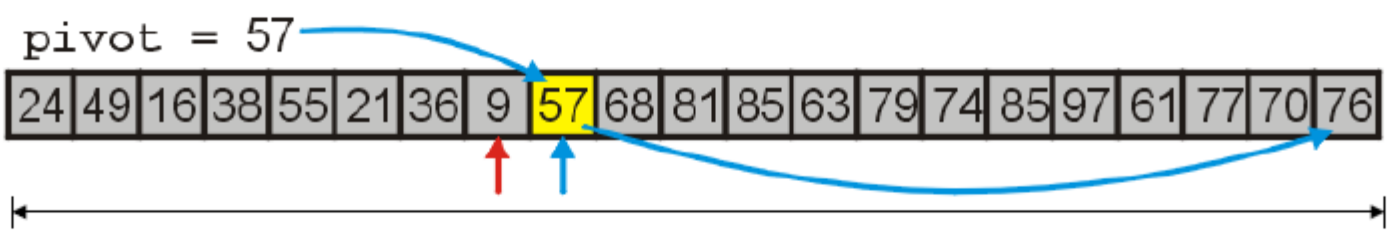
15. Swap 68 and 9



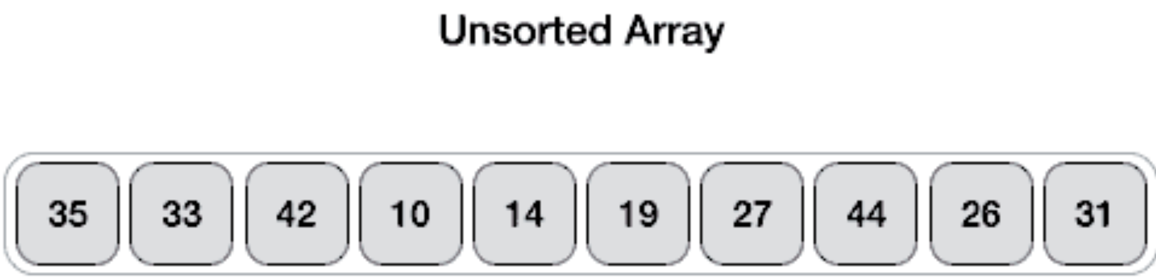
16. search forward until we find 76 ≥ 57
17. search backward until we find 9 < 57
    - The indices are out of order, stop



- Move the first value larger than pivot, i.e., 76, to the last location of the array
- Fill the empty location with the pivot 57
- The pivot is in the correct location



Another example of the partition (animation)



Time complexity of Quick Sort

- Finding the pivot takes time $\Theta(1)$
- Partitioning an array takes time $\Theta(n)$
- Worst case time complexity
  - For each partition, one part has 0 number, the other has n-1 numbers
  - $T(n)= \Theta(n) + T(n-1)$
  - $T(n)= \Theta(n^2)$

Time complexity of Quick Sort

- Best case analysis
  - The best case occurs if the left and right parts are balanced, each has about $n/2$ numbers
  - $T(n) = \Theta(n) + 2T(n/2)$
  - $T(n) = \Theta(n \log n)$

Average time complexity of quick sort

- Consider all cases of the lengths of the two parts
  - Left: 0 number, right: n-1 numbers
  - Left: 1 number, right: n-2 numbers
  - Left: 2 number, right: n-3 numbers
  - …
  - Left: n-1 number, right: 0 numbers
- Assume the probabilities of different cases are equal, i.e., 1/n, we have

$$\mathbf{T}(n) = cn + \frac{1}{n}\sum_{k=0}^{n-1}[\mathbf{T}(k) + \mathbf{T}(n-1-k)], \quad \mathbf{T}(0) = \mathbf{T}(1) = c.$$

$$T(n) = cn + \frac{1}{n}\sum_{k=0}^{n-1}[T(k) + T(n-1-k)] = cn + \frac{2}{n}\sum_{k=0}^{n-1}T(k)$$

$$nT(n) = cn^2 + 2\sum_{k=0}^{n-1}T(k)$$

$$(n-1)T(n-1) = c(n-1)^2 + 2\sum_{k=0}^{n-2}T(k)$$

$$nT(n) - (n-1)T(n-1) = c(2n-1) + 2T(n-1)$$

$$nT(n) = (n+1)T(n-1) + c(2n-1)$$

$$\begin{aligned}
\frac{T(n)}{n+1} &= \frac{T(n-1)}{n} + \frac{c(2n-1)}{n(n+1)} \\
&\leq \frac{T(n-1)}{n} + \frac{c2n}{n(n+1)} \\
&= \frac{T(n-1)}{n} + \frac{2c}{n+1} \\
&= \frac{T(n-2)}{n-1} + \frac{2c}{n} + \frac{2c}{n+1} \\
&\vdots \\
&= \frac{T(1)}{2} + \sum_{i=2}^{n}\frac{2c}{i+1} \\
&\leq 2c\sum_{i=1}^{n-1}\frac{1}{i} \approx 2c\int_1^n \frac{1}{x}\mathrm{d}x = 2c\ln n
\end{aligned}$$

$$T(n) = \Theta(n \log n)$$

Running time comparisons (n=100k)

- Random input

```
100000 numbers to be sorted:
Sort with InsertionSort, Time consumed:  1.438 (seconds)
Sort with bubble sort, Time consumed:    17.360 (seconds)
Sort with SelectionSort, Time consumed:  3.813 (seconds)
Sort with shellSort, Time consumed:      0.016 (seconds)
Sort with heapSort, Time consumed:       0.000 (seconds)
Sort with mergeSort, Time consumed:      0.015 (seconds)
Sort with quickSort, Time consumed:      0.016 (seconds)
```

- The input is already sorted

```
100000 numbers to be sorted:
Sort with InsertionSort, Time consumed:  0.000 (seconds)
Sort with bubble sort, Time consumed:    3.766 (seconds)
Sort with SelectionSort, Time consumed:  3.905 (seconds)
Sort with shellSort, Time consumed:      0.002 (seconds)
Sort with heapSort, Time consumed:       0.005 (seconds)
Sort with mergeSort, Time consumed:      0.004 (seconds)
Sort with quickSort, Time consumed:      0.000 (seconds)
```

- The input is reversely sorted

```
100000 numbers to be sorted:
Sort with InsertionSort, Time consumed:  2.984 (seconds)
Sort with bubble sort, Time consumed:    9.692 (seconds)
Sort with SelectionSort, Time consumed:  3.872 (seconds)
Sort with shellSort, Time consumed:      0.004 (seconds)
Sort with heapSort, Time consumed:       0.005 (seconds)
Sort with mergeSort, Time consumed:      0.005 (seconds)
Sort with quickSort, Time consumed:      0.001 (seconds)
```

Running time comparisons (n=3M)

- Random input

```
3000000 numbers to be sorted:
Sort with shellSort, Time consumed:      1.087 (seconds)
Sort with heapSort, Time consumed:       0.860 (seconds)
Sort with mergeSort, Time consumed:      0.421 (seconds)
Sort with quickSort, Time consumed:      0.334 (seconds)
```

- The input is already sorted

```
3000000 numbers to be sorted:
Sort with shellSort, Time consumed:      0.196 (seconds)
Sort with heapSort, Time consumed:       0.220 (seconds)
Sort with mergeSort, Time consumed:      0.156 (seconds)
Sort with quickSort, Time consumed:      0.033 (seconds)
```

- The input is reversely sorted

```
3000000 numbers to be sorted:
Sort with shellSort, Time consumed:      0.306 (seconds)
Sort with heapSort, Time consumed:       0.217 (seconds)
Sort with mergeSort, Time consumed:      0.179 (seconds)
Sort with quickSort, Time consumed:      0.042 (seconds)
```

Two $\Theta(n)$ sorting algorithms

- Only applicable for special cases, but not general cases
- BinSort
- Radix Sort

BinSort Motivation

- Consider n=5 integers to be sorted:
  - A[5]=1, 5, 4, 9, 2
  - Notice that the maximum number is < 2n = 10
- Allocate an array Bin[10]
- Place A[i] to Bin[A[i]], e.g.,
  - Place A[1] = 5 to Bin[5] by setting Bin[5] = **1**
  - The other values in Bin are **0**

	0	**1**	**1**	**0**	**1**	**1**	**0**	**0**	**0**	**1**
index	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

BinSort Motivation

- A[5]=1, 5, 4, 9, 2

	0	**1**	**1**	**0**	**1**	**1**	**0**	**0**	**0**	**1**
index	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

- BinSort has three steps:

1. Set Bin[j]=0 for 0≤j ≤9
2. Scan array A, set Bin[A[i]]=1 for 0≤i ≤5
3. Scan array Bin from the leftmost to rightmost, if Bin[j] = 1, number j is in array A, and output j

- The output is the sequence:1, 2, 4, 5, 9

Binsort

- A[0, …, n-1],
- Assume that A[i]≥0 and A[i] < c*n, c is a constant, e.g., c = 2
- Allocate an array B with size c*n

```
for (j=0; j<c*n; j++)
    B[j]=0;
for (i=0; i<n; i++)
  ++B[ A[i] ]; // may have duplicate numbers
i=0; // the ith sorted number
for (j=0; j<c*n; j++)//number j appears B[j] times
  for (k=0; k<B[j]; k++, i++)
    A[i] = j;
```

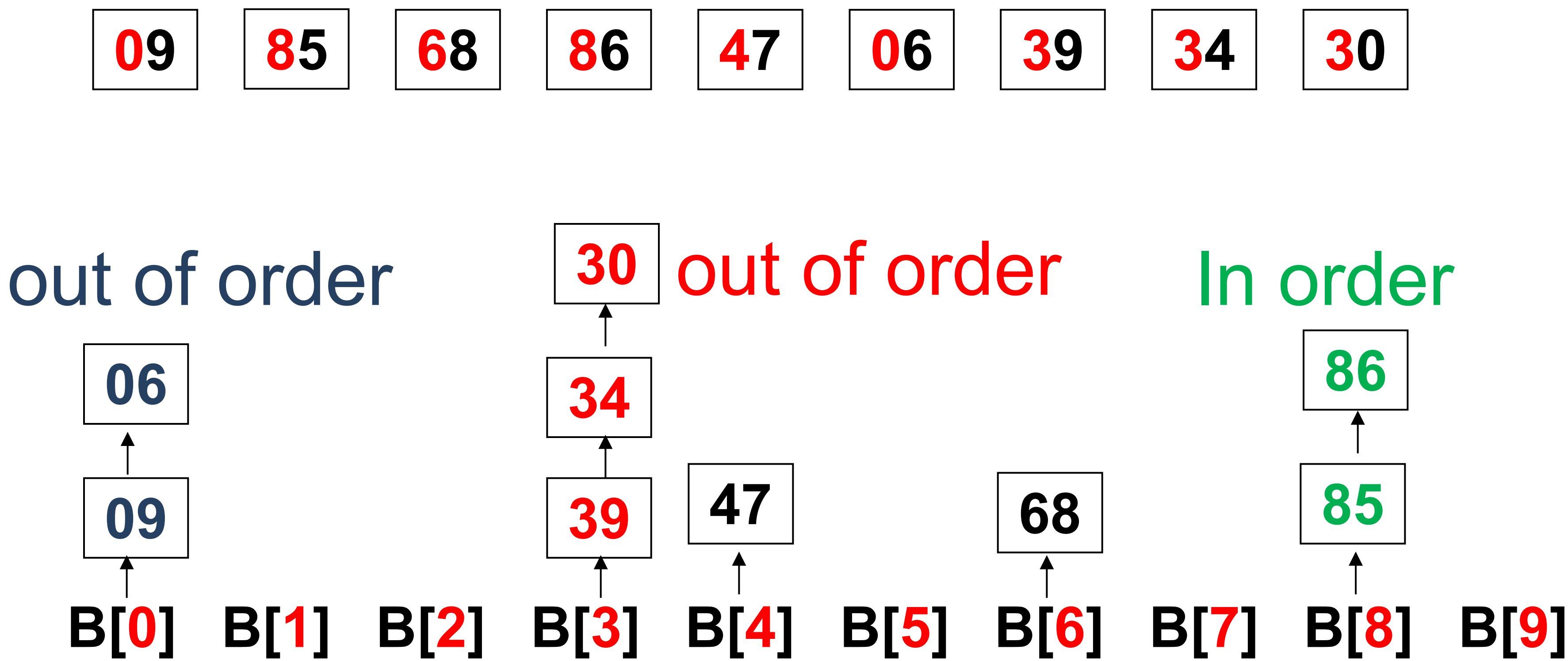
- Time complexity
  - $\Theta(cn) + \Theta(n) + \Theta(cn+n)$
  - $= \Theta(cn) = \Theta(n)$, as c is a constant

The application of BinSort is limited

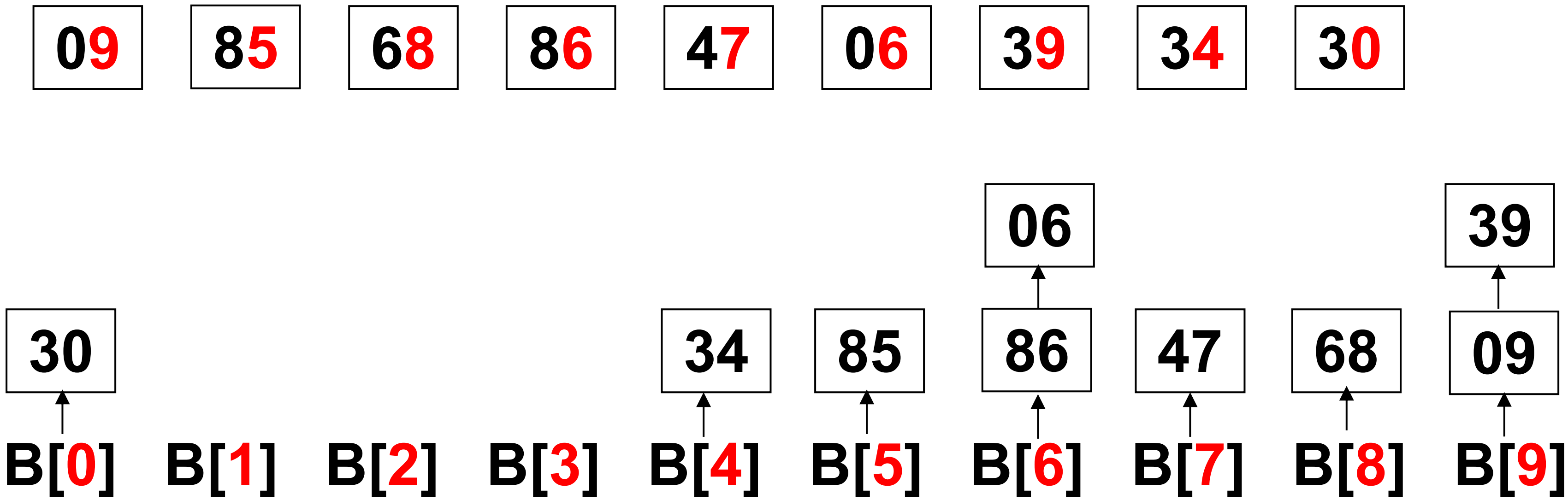
- A[0, …, n-1],
- BinSort is applicable when A[i] < c*n
- Consider another example with n=9 numbers
  - 09, 85, 68, 86, 47, 06, 39, 34, 30
  - The maximum number 86 is about 10 times larger than n, ≥ n^2=81
  - If BinSort is applied, an array B with size 87 ≥ n^2 is needed
  - The time complexity then is in $\Omega(n^2)$

Radix Sort -- Extend BinSort

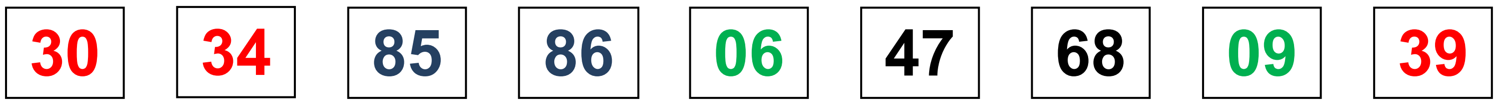
- Some examples of radix or base
- Radix 10: the values of each digit may be 0, 1, 2, …, 9
  - 5_{10}, 16_{10}, 20_{10},…
- Radix 2: each digit is 0 or 1
  - 101_2, 10000_2, 10100_2,…
- Radix 26 (26 letters): a, b, c, …, x, y, z
  - Strings `type’, ‘alpha’, `go’



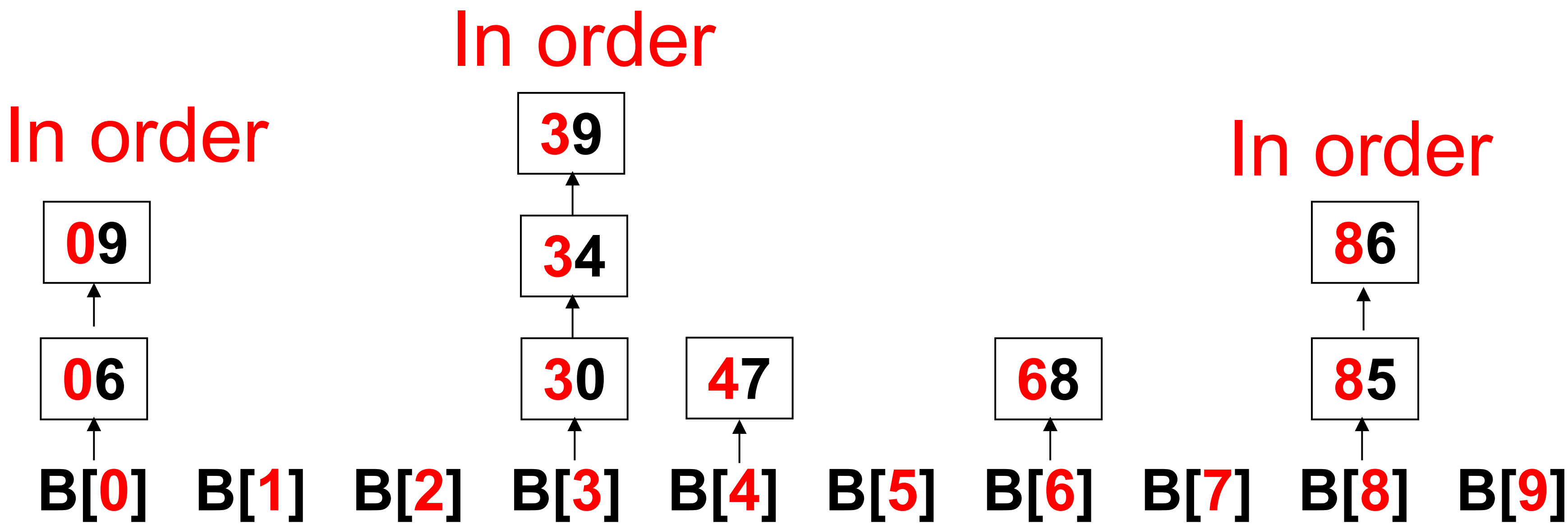
- Each number has two digits
- If we first sort by the highest digit:
- But the numbers in the same bin may be out of order



- What if we first sort by the lowest digit
- We then collect the numbers in the bins
- The numbers with the same highest digit are in order, see the numbers with the same color



30 34 85 86 06 47 68 09 39

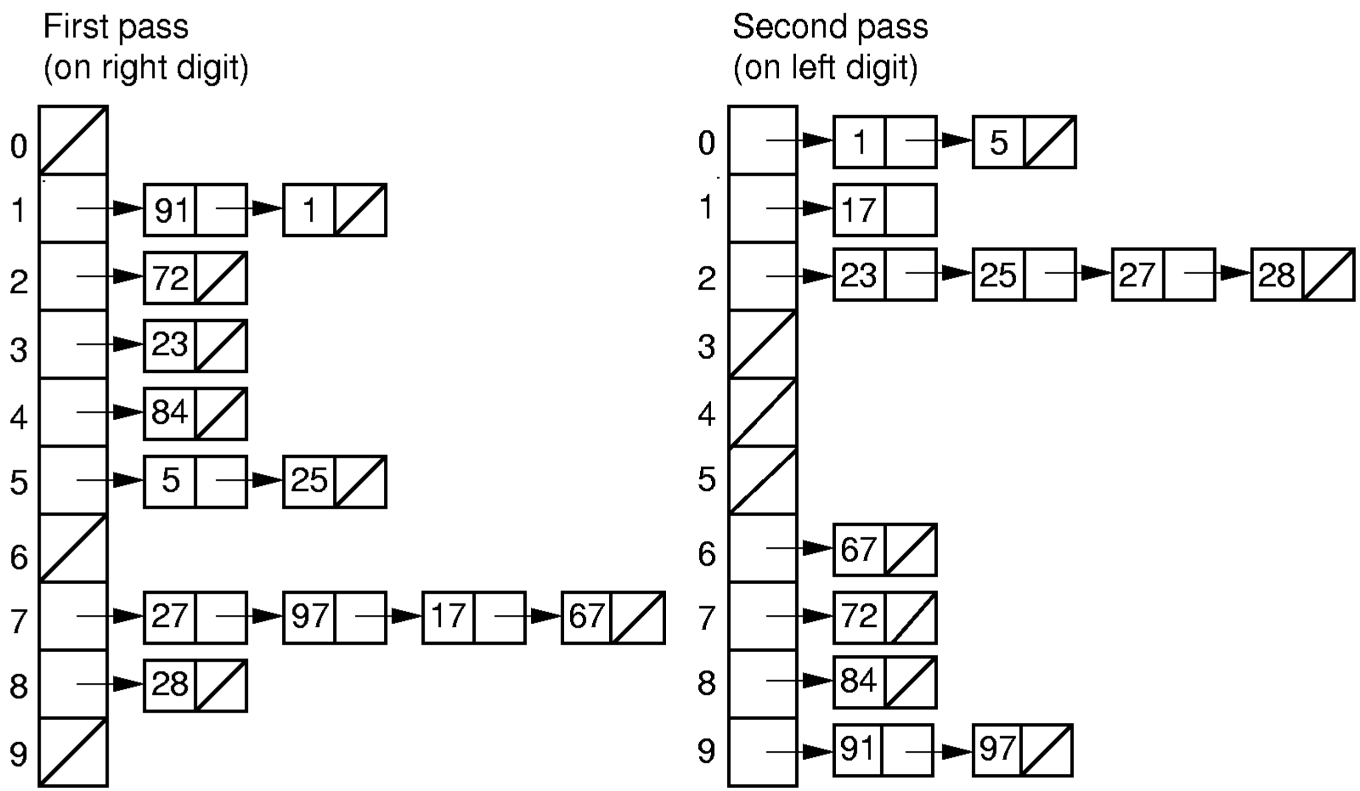


- We then sort the numbers by the highest digit
- Collect the numbers in the bins again
- The numbers are in order now

06 09 30 34 39 47 68 85 89

RadixSort: sort from the lowest digit to the highest digit

Initial List: 27 91 1 97 17 23 84 28 72 5 67 25



Result of first pass: 91 1 72 23 84 5 25 27 97 17 67 28

Result of second pass: 1 5 17 23 25 27 28 67 72 84 91 97

Radix Sort Cost

- Consider n numbers A[0,1, …,n-1] with radix r, each number has no more than k digits
- Has k BinSorts, from the lowest to the highest
- Each sort takes time $\Theta(n+r)$
- Total Cost: $\Theta(k(n+r))$
- If n numbers are distinct, $k >= \log_r n$
- If r is small, e.g., r=2, radixSort is in $\Theta(n \log n)$
- We usually use large values of r, e.g., r=1K, 1M, or even, n

Running time comparisons (n=3M)

- random input
- RadixSort is faster if r is larger, 10 ≤r≤100k
- But does not improve any more when r approaches to n

```
3000000 numbers to be sorted:
Sort with shellSort, Time consumed:      1.062 (seconds)
Sort with heapSort, Time consumed:       0.953 (seconds)
Sort with mergeSort, Time consumed:      0.438 (seconds)
Sort with quickSort, Time consumed:      0.328 (seconds)
Sort with radixSort (r=10), Time consumed:        0.313 (seconds)
Sort with radixSort (r=100), Time consumed:       0.156 (seconds)
Sort with radixSort (r=1000), Time consumed:      0.141 (seconds)
Sort with radixSort (r=10000), Time consumed:     0.093 (seconds)
Sort with radixSort (r=100000), Time consumed:    0.078 (seconds)
Sort with radixSort (r=1000000), Time consumed:   0.079 (seconds)
```

Running time comparisons (n=3M)

- The input is already sorted
  - quickSort is faster than radixSort

```
3000000 numbers to be sorted:
Sort with shellSort, Time consumed:      0.172 (seconds)
Sort with heapSort, Time consumed:       0.234 (seconds)
Sort with mergeSort, Time consumed:      0.156 (seconds)
Sort with quickSort, Time consumed:      0.031 (seconds)
Sort with radixSort (r=10), Time consumed:       0.329 (seconds)
Sort with radixSort (r=100), Time consumed:      0.156 (seconds)
Sort with radixSort (r=1000), Time consumed:     0.156 (seconds)
Sort with radixSort (r=10000), Time consumed:    0.141 (seconds)
Sort with radixSort (r=100000), Time consumed:   0.109 (seconds)
Sort with radixSort (r=1000000), Time consumed:  0.063 (seconds)
```

- The input is reversely sorted

```
3000000 numbers to be sorted:
Sort with shellSort, Time consumed:      0.281 (seconds)
Sort with heapSort, Time consumed:       0.234 (seconds)
Sort with mergeSort, Time consumed:      0.172 (seconds)
Sort with quickSort, Time consumed:      0.032 (seconds)
Sort with radixSort (r=10), Time consumed:       0.313 (seconds)
Sort with radixSort (r=100), Time consumed:      0.156 (seconds)
Sort with radixSort (r=1000), Time consumed:     0.156 (seconds)
Sort with radixSort (r=10000), Time consumed:    0.156 (seconds)
Sort with radixSort (r=100000), Time consumed:   0.110 (seconds)
Sort with radixSort (r=1000000), Time consumed:  0.078 (seconds)
```

The limitation of RadixSort

- Only applicable to sorting integers
- But inapplicable for
  - real numbers
  - Strings has arbitrarily length
    - E.g., short string \`a’, long string \`dfdfldlfdfdfldjfdlfjslfjsdfdfdfdoojll’
  - …

Lower Bound for Sorting

- We would like to know a lower bound for all possible sorting algorithms
- Sorting is $O(n \log n)$ (average, worst cases) because we know algorithms with this upper bound, e.g., MergeSort or HeapSort
- Sorting takes $\Omega(n)$ time, as each number must be accessed at least once
- Is there any one better than $\Theta(n \log n)$?
- It is proved that sorting is $\Omega(n \log n)$
- MergeSort and HeapSort are asymptotically optimal !

Chapter III-8. File Processing and External Sorting

Primary vs. Secondary Storage

- Primary storage: Main memory (RAM)
  - volatile, i.e., data is lost if powered off
  - Usually a few GB
  - Expensive (unit: $/MB), fast

- Secondary Storage: Peripheral devices
  - Hard Disk, Solid State Drive (SSD), USB, CD, Tape,…
  - Non-volatile
  - Hundreds of GB, or TB
  - Cheap and slow

Performance Comparisons (typical values)

	Sequential read	seq. write	Random read	Random write
RAM	5 GB/s	4 GB/s	300 MB/s	250 MB/s
Hard Disk	80 MB/s	80 MB/s	0.3 MB/s	0.5 MB/s
SSD	200 MB/s	80 MB/s	25 MB/s	70 MB/s

- Performance of hard disks is terribly poor for random read and write

Golden Rule of File Processing

- Minimize the number of disk accesses!
  - Arrange information so that you get what you want with few disk accesses
    - Store data on adjacent tracks, rather than randomly
  - Arrange information to minimize future disk accesses
    - Cache

External Sorting

- Problem: Sorting data sets too large to fit into main memory.
  - Assume data are stored on disk drive.
- To sort, portions of the data must be brought into main memory, processed, and returned to disk.
- An external sort should minimize disk accesses.

Model of External Computation

- As sequential access is much more efficient than random access to the file
  - adjacent logical blocks of the file must be physically adjacent.

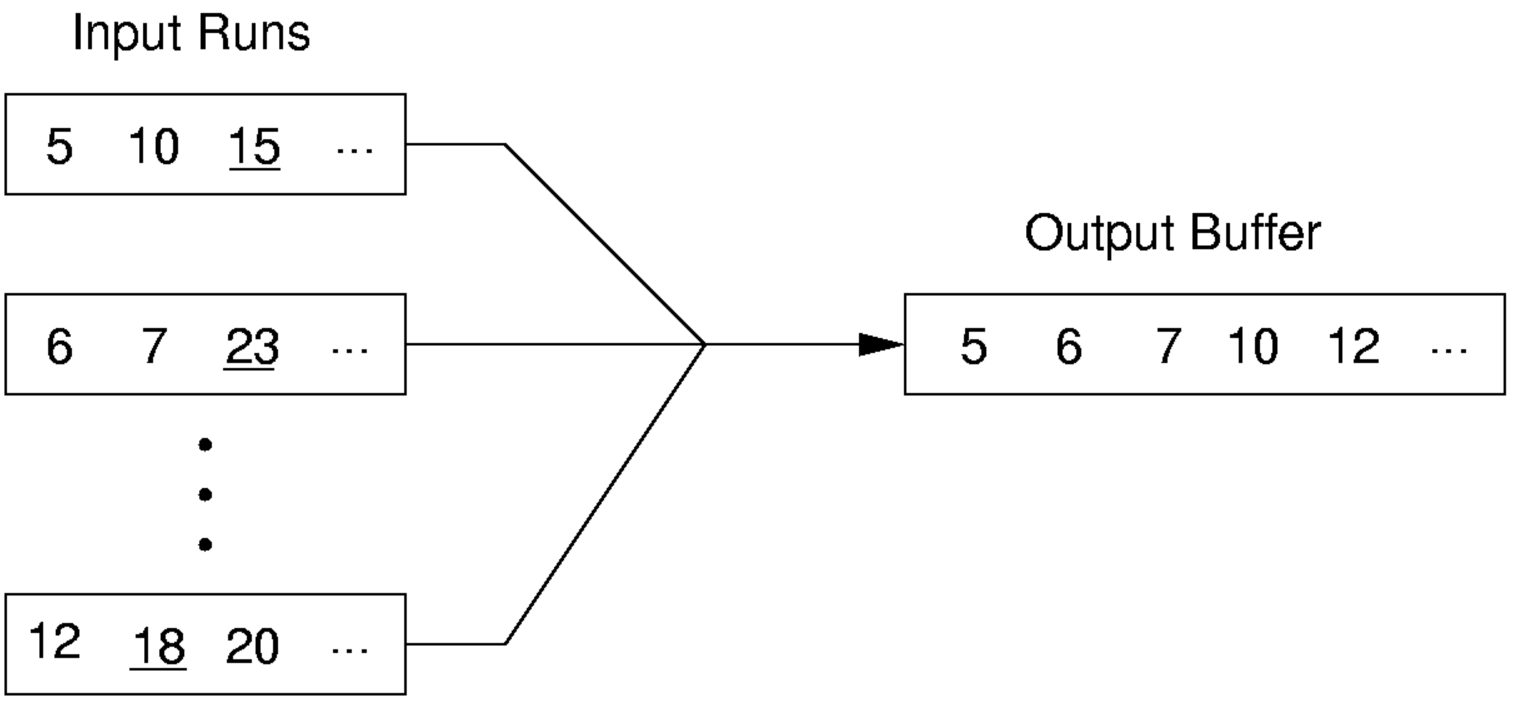
External Sorting

- Three Steps:

1. Break a large file into multiple small initial blocks, so that each block can be fit into memory
   - E.g., break a 10 GB file into 10 blocks with each being 1 GB
2. Sorting the blocks by a fast internal sorting algorithm one by one, and write back to hard disks
3. Merge the sorted blocks together to form a single sorted file.

Multiway Merge

- Merge multiple blocks together, not just two blocks as the internal MergeSort



Homework 3

- See course webpage
- Deadline: midnight before next lecture
- Submit to: cs_scu@foxmail.com
- File name format:
  - CS311_Hw3_yourID_yourLastName.doc (or .pdf)